

A deformation of $AdS_5 \times S^5$

Jerome P. Gauntlett^{1*}, Jan B. Gutowski² and Nemani V. Suryanarayana¹

¹*Perimeter Institute for Theoretical Physics
Waterloo, ON, N2J 2W9, Canada
E-mail: jgauntlett, vnemani@perimeterinstitute.ca*

²*Mathematical Institute, Oxford University,
Oxford, OX1 3LB, U.K.
E-mail: gutowski@maths.ox.ac.uk*

Abstract

We analyse a one parameter family of supersymmetric solutions of type IIB supergravity that includes $AdS_5 \times S^5$. For small values of the parameter the solutions are causally well-behaved, but beyond a critical value closed timelike curves (CTC's) appear. The solutions are holographically dual to $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on a non-conformally flat background with non-vanishing R -currents. We compute the holographic energy-momentum tensor for the spacetime and show that it remains finite even when the CTC's appear. The solutions, as well as the uplift of some recently discovered AdS_5 black hole solutions, are shown to preserve precisely two supersymmetries.

*On leave from: Blackett Laboratory, Imperial College, London, SW7 2BZ, U.K.

1 Introduction

A powerful way to generalise the basic examples of the *AdS/CFT* correspondence in string and M-theory [1] is to use gauged supergravities. For example, the bosonic sector of minimal gauged supergravity in D=5 [2] can be obtained via a consistent truncation of the Kaluza-Klein reduction of type IIB supergravity on a five-sphere [3], which implies that any bosonic solution of the gauged supergravity can be uplifted to obtain a solution of type IIB supergravity. The AdS_5 vacuum solution uplifts to the $AdS_5 \times S^5$ solution, which is dual to four-dimensional $\mathcal{N} = 4$ super-symmetric Yang-Mills theory, while non-vacuum solutions uplift to solutions containing a deformed five-sphere, which can correspond to various deformations of this CFT.

In [4] a one-parameter family of supersymmetric solutions of D=5 minimal gauged supergravity were constructed, which includes the AdS_5 vacuum solution when the parameter is set to zero. We will show here that the family of solutions are asymptotically locally AdS_5 , and hence the corresponding uplifted type IIB solutions can be interpreted as being dual to $\mathcal{N} = 4$ supersymmetric Yang-Mills theory living on the non-trivial conformal boundary, which we show is not conformally flat. Since the gauged supergravity solutions have non-vanishing $U(1)$ gauge-fields, this implies that R -symmetry currents are also non-vanishing in the dual picture. Note that this interpretation was also discussed in [5], where the solutions were further generalised (see also [6]).

An interesting feature of the solutions of [4] is that closed time-like curves (CTCs) appear when the parameter is larger than a critical value. There have been several recent investigations aiming to understand the role, if any, of CTC's in string theory. This activity was catalysed by the discovery of the supersymmetric Gödel solution of minimal *ungauged* supergravity in D=5 [7]. These solutions are homogeneous and preserve twenty supersymmetries when uplifted to provide a solution of D=11 supergravity. In [8] it was suggested that the physics of these spacetimes might be encoded holographically in observer dependent screens of the type discussed in [9]. Somewhat related to this idea is the possibility that only a region of the Gödel spacetime makes sense in string theory: the rest of the spacetime, including the CTCs, should be excised and replaced by another spacetime in order to get a physically sensible background [10, 11, 12, 13] (for related work see, for example, [14]-[25]). However, it is fair to say that the matter is not yet settled.

It is natural to wonder if standard *AdS/CFT* holography can provide a new perspective on CTCs. For example, it would be interesting if there are supersymmetric solutions of string/M-theory that are asymptotically locally *AdS* with CTCs confined to the bulk. Such solutions might then be dual to a CFT on the boundary which could then provide a precise description, in principle, of string/M-theory propagating on the spacetime with CTCs. In any case, we will show here that the solutions of [4] are not in this class: for values of the parameter when the CTC's appear in the bulk, they also appear on the boundary.

Nevertheless, one might boldly assume that the duality is still valid for our solutions for supercritical values of the parameter. The supergravity solution should then be dual to $\mathcal{N} = 4$ SYM on a background containing CTCs. Now from intuition gar-

nered from analysing quantum fields on curved spacetimes, one might expect that the energy-momentum tensor of the field theory to diverge on such a spacetime. Indeed, the energy-momentum tensor of a scalar field propagating on a one-parameter family of solutions that includes the four-dimensional Gödel spacetime [26] was calculated using zeta-function regularisation in [27]. As the parameter is varied, it was shown that the energy-momentum tensor becomes divergent precisely at the onset of CTCs.

Here we will calculate the holographic energy-momentum (EM) tensor for our family of solutions using the beautiful procedure of Balasubramanian and Kraus [28]. Somewhat surprisingly, we find that it is finite for all values of the parameter, which may be construed as a hint that the duality is indeed valid even when the CTCs are present. However, it is only a hint. If it turns out that the supergravity solutions with CTC's are simply not physical, it would clearly be very interesting to identify the bound on the parameter directly in $\mathcal{N} = 4$ super-Yang-Mills theory. Perhaps it represents a kind of unitary bound as, for example, in the case of the BMPV black hole [29] where the onset of CTCs has been argued to correspond to a unitary bound in the dual CFT [30].

By construction, the D=5 solutions preserve two supersymmetries, or 1/4 of the D=5 supersymmetry [4]. Now it is expected that the full supersymmetric D=5 supergravity theory can be obtained via consistent truncation of the Kaluza-Klein reduction of type IIB supergravity on a five-sphere (this was only shown for the bosonic sector in [3]), and hence it is expected that the uplifted solutions will preserve at least two supersymmetries or 1/16 of type IIB supergravity. We show here that they preserve precisely two supersymmetries and present explicit expressions for the Killing spinors. This is in marked contrast to the Gödel solution of minimal ungauged supergravity in D=5, which preserves eight supersymmetries in D=5 and yet, as remarked above, twenty supersymmetries when uplifted to obtain a solution of D=11 supergravity.

Our method of determining the supersymmetry, mostly using a computer algebraic package, is easily adapted to determine the preserved type IIB supersymmetry of any uplifted solution of minimal gauged supergravity in D=5. Recently, black hole solutions that are asymptotically globally AdS_5 were found [31] (similar solutions were found for non-minimal D=5 gauged supergravity in [32]), and we show here that they also preserve just two supersymmetries when uplifted to get type IIB solutions. This is very important in seeking a microscopic interpretation of the entropy of these black holes: our result implies that the entropy must correspond to states in $\mathcal{N} = 4$ super-Yang-Mills theory preserving precisely 1/16 supersymmetry.

The plan of the paper is as follows. Section 2 analyses the D=5 solution and calculates the energy-momentum tensor. Section 3 analyses the uplifted type IIB solution and determines the amount of preserved supersymmetry. Analogous results for the uplifted AdS_5 black hole solutions are presented in section 4. The paper contains three appendices.

2 The five-dimensional solution

We start with the family of solutions of minimal five-dimensional gauged supergravity found in Ref.[4] whose metric, written in new co-ordinates, is

$$ds_5^2 = - \left(dt + \frac{r^2}{2l} \sigma_3^L + \frac{fr^2}{V(r)} \sigma_1^L \right)^2 + \frac{dr^2}{V(r)} + \frac{r^2}{4} [(\sigma_1^L)^2 + (\sigma_2^L)^2 + V(r) (\sigma_3^L)^2] \quad (2.1)$$

where $V(r) = 1 + r^2/l^2$ and the right-invariant 1-forms on the three-sphere, σ_i^L , are given by¹

$$\begin{aligned} \sigma_1^L &= \sin \phi d\theta - \sin \theta \cos \phi d\psi, \\ \sigma_2^L &= \cos \phi d\theta + \sin \theta \sin \phi d\psi, \\ \sigma_3^L &= d\phi + \cos \theta d\psi. \end{aligned} \quad (2.2)$$

(Note that χ , $\mathcal{F}_1, \mathcal{F}_2$ of [4] are related to l, f via $l = \sqrt{12}/\chi$ and $f = \mathcal{F}_1/4$ with $\mathcal{F}_2 = 0$.) This metric is supported by a $U(1)$ gauge field whose field strength is given by:

$$F^{(2)} = \frac{\sqrt{3}}{2} f d \left(\frac{r^2}{V} \sigma_1^L \right) = \frac{\sqrt{3}}{2} f \left(\frac{2r}{V^2} dr \wedge \sigma_1^L - \frac{r^2}{V} \sigma_2^L \wedge \sigma_3^L \right). \quad (2.3)$$

Note that the Killing vector ∂_t is timelike and that there are additional Killing vectors that generate an $SU(2)_R$ group of isometries. The solution preserves two supersymmetries, i.e. 1/4 of the supersymmetry.

It is easy to see that when $f = 0$, the metric (2.1) is just that of AdS_5 . To see this, set $f = 0$ and introduce the new co-ordinate

$$\tilde{\phi} = \phi - 2t/l \quad (2.4)$$

so that the metric simplifies to

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + \frac{r^2}{4} [(\tilde{\sigma}_1^L)^2 + (\tilde{\sigma}_2^L)^2 + (\tilde{\sigma}_3^L)^2] \quad (2.5)$$

where $\tilde{\sigma}_i^L$ are as in (2.2) with ϕ replaced with $\tilde{\phi}$. This is AdS_5 in global coordinates. Thus the real parameter f is a measure of the deformation of the metric away from that of AdS_5 . Note that performing the same coordinate change when $f \neq 0$ leads to time-dependent metric components.

It was shown in [4] that the spacetime has closed time-like curves when $f^2 l^2$ is large. It was shown in [6], more precisely, that the metric has closed time-like curves when $f^2 > 1/(4l^2)$, and that they are absent when $f^2 \leq 1/(4l^2)$. Moreover, the closed time-like curves go through all points.

Next observe that the metric is locally asymptotically AdS_5 . This can be seen by evaluating the Riemann tensor $R_{ab}{}^{cd}$. One finds:

$$R_{ab}{}^{cd} = -\frac{1}{l^2} (\delta_a^c \delta_b^d - \delta_a^d \delta_b^c) + \mathcal{O}\left(\frac{1}{r}\right) \quad (2.6)$$

¹These are dual to right-invariant vector fields that generate left actions, and hence the superscript L .

in the limit as $r \rightarrow \infty$. Thus one expects on general grounds that the holographic dual of the supergravity (and string) theory on this background should be dual to four-dimensional $\mathcal{N} = 4$ $SU(N)$ SYM on the boundary of this geometry. The non-vanishing $U(1)$ gauge-fields indicate that there are non-vanishing R -symmetry currents in the dual theory. This was also discussed in [5].

We note that

$$\begin{aligned} *_{(5)} F^{(2)} = 2\sqrt{3}f \Big[& \frac{r^2}{4V} dt \wedge \sigma_2^L \wedge \sigma_3^L - \frac{r}{2V^2} dt \wedge dr \wedge \sigma_1^L \\ & - \frac{r^3}{4lV^2} dr \wedge \sigma_1^L \wedge \sigma_3^L + \frac{fr^4}{4V^2} \sigma_1^L \wedge \sigma_2^L \wedge \sigma_3^L \Big] . \end{aligned} \quad (2.7)$$

In particular if we integrate this over a surface Σ defined at fixed r, t as $r \rightarrow \infty$ we get

$$\int_{\Sigma} *_{(5)} F^{(2)} = 8\sqrt{3}\pi^2 f^2 l^4 \quad (2.8)$$

indicating that the solution carries electric charge.

2.1 The four-dimensional boundary geometry

As $r \rightarrow \infty$ the metric (2.1) takes the form

$$ds^2 = \frac{l^2}{r^2} dr^2 + r^2 (ds_{bdy}^2) + \mathcal{O}(1) \quad (2.9)$$

where the boundary metric is defined as

$$ds_{bdy}^2 = -\frac{1}{l} (dt + fl^2 \sigma_1^L) \sigma_3^L + \frac{1}{4} \sigma_i^L \sigma_i^L . \quad (2.10)$$

This is a regular metric and when $f = 0$ we get the standard metric on $\mathbb{R} \times S^3$ after employing the coordinate transformation (2.4). One can calculate scalar curvature invariants and we find that it has constant Ricci scalar curvature given by $R = 6$ and $R_{\mu\nu} R^{\mu\nu} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 12$. In particular we conclude that the Euler invariant, $E \equiv R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2$, and the Weyl invariant $C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 2R_{\mu\nu} R^{\mu\nu} + R^2/3$ both vanish. Note that the boundary metric is not Einstein, and since the Weyl tensor does not vanish, when $f \neq 0$, it is not conformally flat.

Note that the killing vector $\partial/\partial t$ is now null and that the metric maintains the right- $SU(2)_R$ isometries. Note also that the condition for closed time-like curves on the boundary is the same as that in the bulk.

2.2 The energy-momentum tensor

We now calculate the boundary energy-momentum tensor for this system². From [28] we have³

$$T_{\mu\nu} = \lim_{r \rightarrow \infty} \frac{1}{8\pi G} r^2 \left[\Theta_{\mu\nu} - \Theta \gamma_{\mu\nu} - \frac{3}{l} \gamma_{\mu\nu} + \frac{l}{2} G_{\mu\nu} \right] \quad (2.11)$$

where the extrinsic curvature

$$\Theta_{\mu\nu} = \nabla_\mu \hat{n}_\nu \quad (2.12)$$

is calculated with the unit normal $\hat{n}^\mu = (0, -V^{1/2}, 0, 0, 0)$ and the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R \gamma_{\mu\nu}$ is calculated for the 4-d metric induced on a slice of constant r . The factor of r^2 appearing in (2.11) is to obtain the energy-momentum tensor on the boundary geometry (2.10). One finds

$$\begin{aligned} 8\pi GT &= \frac{l}{2} dt^2 - \frac{l^2}{8} dt(\sigma_3^L - 24fl\sigma_1^L) + \frac{l^3}{32}(1 + 80(fl)^2)(\sigma_1^L)^2 \\ &+ \frac{l^3}{32}(\sigma_2^L)^2 + \frac{l^3}{32}(1 + 64(fl)^2)(\sigma_3^L)^2 - \frac{13fl^4}{8}\sigma_1^L\sigma_3^L. \end{aligned} \quad (2.13)$$

We have directly checked that this energy-momentum tensor satisfies the conservation law $\nabla^\mu T_{\mu\nu} = 0$ with respect to the boundary metric (2.10), as expected. Furthermore, the trace with respect to the boundary metric also vanishes, $T_\mu{}^\mu = 0$, which is consistent with the vanishing of the Euler and Weyl invariants of the boundary metric. Finally, we observe that when $f = 0$ we obtain the correct result for AdS_5 . In particular, setting $f = 0$ and employing the co-ordinate transformation (2.4) we get,

$$8\pi GT = \frac{3l}{8} dt^2 + \frac{l^3}{32} [(\tilde{\sigma}_1^L)^2 + (\tilde{\sigma}_2^L)^2 + (\tilde{\sigma}_3^L)^2] . \quad (2.14)$$

To further analyse the EM tensor, we consider its components with respect to an orthonormal frame for the boundary metric. In particular, using the frame

$$\begin{aligned} e^0 &= \frac{1}{l} dt + fl\sigma_1^L, & e^1 &= \frac{1}{2}\sigma_1^L, \\ e^2 &= \frac{1}{2}\sigma_2^L, & e^3 &= \frac{1}{2}\sigma_3^L - \frac{1}{l} dt - fl\sigma_1^L \end{aligned} \quad (2.15)$$

we find that the energy-momentum tensor has components

$$8\pi GT_{ab} = \frac{l^3}{8} \begin{bmatrix} 3 + x^2 & -x & 0 & x^2 \\ -x & 1 & 0 & -3x \\ 0 & 0 & 1 & 0 \\ x^2 & -3x & 0 & 1 + x^2 \end{bmatrix} \quad (2.16)$$

²After this paper was submitted to the arXive, a revised version of [5] appeared where this was independently calculated [5].

³Note that here we do not need any contributions from the gauge-field of the type discussed in [33] to cancel any divergences.

where $x = 8fl$. The most striking feature of this energy-momentum tensor is that it is not divergent for any value of the deformation parameter. Moreover, it does not undergo any particular transformation as the CTC's appear when $x = 4$.

To gain some further insight, we first recall (e.g. section 4.3 of [34]) that any energy-momentum tensor in four dimensions is in one of four canonical classes, depending on whether $T_\mu{}^\nu$ has one timelike eigenvector, a double null eigenvector, a triple null eigenvector, or neither a time-like nor a null eigenvector, respectively. Indeed for $x < x_c$, where $x_c \approx 2.515534$, there is one time-like and three spacelike eigenvectors and we are in the first class of [34]. Moreover, one can check that the weak energy condition holds. For $x > x_c$ there are just two space-like eigenvectors and we are in the fourth class of [34]. In [34] it is stated that there are no observed fields with energy-momentum tensors of this form, so this may be an indication that the supergravity solutions are not physical, which would then also exclude the solutions with CTC's. Of course the EM tensor above is the expectation value in the quantum theory and moreover corresponds to large 't Hooft coupling, so it seems quite plausible to us that the solutions are physical all the way up to the onset of CTC's at $x = 4$.

We should also point out that there are some well known ambiguities in the energy-momentum tensor computed using the counterterm subtraction method of [28]. Firstly, one is free to add the following local gravitational counterterms to the boundary action:

$$\frac{l^3}{8\pi G} \int_{\partial\mathcal{M}_r} d^4x \sqrt{-\gamma} [\alpha R^2 + \beta R^{\mu\nu} R_{\mu\nu} + \gamma R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}] , \quad (2.17)$$

where α, β and γ are constants. These terms contribute [37]

$$\mathcal{H}_{\mu\nu} = \frac{l^3}{4\pi G} [\alpha {}^{(1)}H_{\mu\nu} + \beta {}^{(2)}H_{\mu\nu} + \gamma H_{\mu\nu}] \quad (2.18)$$

to the boundary EM tensor. We have presented the explicit expressions of these quantities for our solution in appendix C. The fact that the Gauss-Bonnet combination $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + R^2 - 4R_{\mu\nu} R^{\mu\nu}$ is topological in four-dimensions implies that $H_{\mu\nu} = -{}^{(1)}H_{\mu\nu} + 4 {}^{(2)}H_{\mu\nu}$. Therefore we may always set one of α, β, γ to zero, say $\gamma = 0$. If the boundary metric $\gamma_{\mu\nu}$ is conformally flat, $C_{\mu\nu\rho\sigma} = 0$, we further have ${}^{(2)}H_{\mu\nu} = (1/3){}^{(1)}H_{\mu\nu}$ which reduces the ambiguity down to a single parameter. Usually this remaining ambiguity is fixed by demanding that the EM tensor of the boundary CFT does not have any trace anomaly proportional to $\square R$. If $\gamma_{\mu\nu}$ is not conformally flat $\mathcal{H}_\mu{}^\mu = 0$ requires $3\alpha + \beta = 0$, which then leaves a single free parameter.

Besides the finite gravitational counterterms, there is also the following counterterm depending on the gauge field:

$$-\frac{l\delta}{8\pi G} \int_{\partial\mathcal{M}_r} d^4x \sqrt{-\gamma} F_{\mu\nu}^{(2)} F^{(2)\mu\nu} \quad (2.19)$$

where $F^{(2)}$ is the two-form field-strength at fixed r and δ is another constant. These and related contributions have been considered recently in a different context in [38, 39]. We have also listed the contributions of these finite counterterms to the EM tensor of our boundary CFT in appendix C.

2.3 Conserved Quantities

Having obtained the energy-momentum tensor we can determine the conserved charges corresponding to the Killing vectors of the boundary metric as explained in [28]. The general prescription is to first pick a spacelike 3-surface Σ on the boundary with metric σ_{ab} . Let u^μ be a timelike (not necessarily Killing) vector normal to Σ and let ξ^μ be a Killing vector of the boundary metric. Then the conserved quantity associated to this Killing vector is:

$$Q_\xi = \int_\Sigma d^3x \sqrt{\sigma} (u^\mu \xi^\nu T_{\mu\nu}) . \quad (2.20)$$

Consider the surface Σ defined by $t=\text{constant}$. We restrict to the case of no CTC's, $4f^2l^2 < 1$, in which case Σ is indeed spacelike. The unit normal u , written as a one-form, is equal to $-dt/(l\sqrt{1-4l^2f^2})$. A straightforward calculation reveals that the conserved quantities associated with the $SU(2)$ Killing-vectors ξ_i^R all vanish (for explicit expressions for ξ_i^R see, e.g. appendix A of [35]). Similarly, the conserved quantity for the null Killing vector $\xi = \frac{\partial}{\partial t}$ is given by

$$Q_{\frac{\partial}{\partial t}} = \frac{1}{8\pi G} \frac{l^2(3 + 32l^2f^2)\pi^2}{4} . \quad (2.21)$$

It is interesting to note that when written in the co-ordinates (2.4) the null Killing-vector becomes $\frac{\partial}{\partial t} - (2/l)\xi_3^L$, where $\xi_3^L = \frac{\partial}{\partial \phi}$. Now when $f = 0$, the five-dimensional solution is AdS_5 , and both $\frac{\partial}{\partial t}$ (in the new co-ordinates) and ξ_3^L are Killing-vectors, with the former generating global time translations. Thus when $f = 0$ the conserved quantity $Q_{\frac{\partial}{\partial t}}$ is simply the linear combination of charges $M - (1/l)(J_1 + J_2)$, where M is the mass and J_i are the two $SO(4)$ Casimirs. Since $J_i = 0$ for global AdS_5 , when $f = 0$ we recover the mass of global AdS_5 , which is naturally interpreted as the Casimir energy of $\mathcal{N} = 4$ super-symmetric Yang-Mills theory on $\mathbb{R} \times S^3$ [28]. When $f \neq 0$, neither M nor $(J_1 + J_2)$ are separately conserved quantities, but the linear combination $Q_{\frac{\partial}{\partial t}}$ is.

Finally, we comment on the effect of the counterterms (2.17) and (2.19) on the conserved charges. We find that the gauge-field counterterm (2.19) does not affect the $Q_{\xi_i^R}$ or $Q_{\frac{\partial}{\partial t}}$. The gravitational counter-terms (2.17) leave the charges $Q_{\xi_i^R}$ invariant, but they do modify $Q_{\frac{\partial}{\partial t}}$. In particular, we find, when $\gamma = 0$,

$$\Delta Q_{\frac{\partial}{\partial t}} = -\frac{1}{8\pi G} 24l^2(3\alpha + \beta)\pi^2 . \quad (2.22)$$

It is interesting to note that this vanishes when $3\alpha + \beta = 0$, which is precisely the condition required for the EM tensor not to have any trace anomaly proportional to $\square R$.

3 The IIB solution and its supersymmetry

Any bosonic solution of the minimal gauged supergravity in D=5 can be uplifted to obtain a solution of ten-dimensional type IIB supergravity, with the only non-

vanishing fields being the metric and the self-dual five-form using⁴ the formulae in [3]. Such uplifted solutions are expected to preserve at least as many supersymmetries as the solution in D=5.

Specifically, starting from a D=5 solution with metric ds_5^2 and field strength $F^{(2)} = dA$, the uplifted type IIB supergravity solution is given by:

$$\begin{aligned} ds_{10}^2 &= ds_5^2 + l^2 \sum_{i=1}^3 \left[(d\mu_i)^2 + \mu_i^2 \left(d\xi_i + \frac{2}{l\sqrt{3}} A \right)^2 \right], \\ F^{(5)} &= (1 + *_{(10)}) \left[-\frac{4}{l} \text{vol}_{(5)} + \frac{l^2}{\sqrt{3}} \sum_{i=1}^3 d(\mu_i^2) \wedge d\xi_i \wedge *_{(5)} F^{(2)} \right] \end{aligned} \quad (3.1)$$

where $\mu_1 = \sin \alpha$, $\mu_2 = \cos \alpha \sin \beta$, $\mu_3 = \cos \alpha \cos \beta$ with $0 \leq \alpha \leq \pi/2$, $0 \leq \beta \leq \pi/2$, $0 \leq \xi_i \leq 2\pi$ and together they parametrise S^5 . Note that we define the Hodge star of a p -form ω in n -dimensions as $*_{(n)} \omega_{i_1 \dots i_{n-p}} = \frac{1}{p!} \epsilon_{i_1 \dots i_{n-p}}^{j_1 \dots j_p} \omega_{j_1 \dots j_p}$, with $\epsilon_{0123456789} = 1$ and $\epsilon_{01234} = 1$ in an orthonormal frame. Finally, $\text{vol}_{(5)} = \epsilon$ is the volume five-form of the metric (2.1).

Any uplifted D=5 solution satisfies the type IIB Einstein equations

$$R_{mn} - \frac{1}{96} F_{mp_1 p_2 p_3 p_4}^{(5)} F_{n}{}^{p_1 p_2 p_3 p_4} = 0. \quad (3.2)$$

The five-form is obviously self-dual, $F^{(5)} = *_{(10)} F^{(5)}$, and one can show that it also satisfies the Bianchi identity

$$dF^{(5)} = 0. \quad (3.3)$$

We have directly checked that uplifting the solution (2.1), (2.3) solves these IIB equations of motion.

In our conventions the conditions for preserved IIB supersymmetry are given by

$$\begin{aligned} \hat{\nabla}_m \epsilon &\equiv D_m \epsilon + \frac{i}{1920} \Gamma^{n_1 n_2 n_3 n_4 n_5} \Gamma_m F_{n_1 n_2 n_3 n_4 n_5}^{(5)} \epsilon \\ &= D_m \epsilon + \frac{i}{192} \Gamma^{n_1 n_2 n_3 n_4} F_{mn_1 n_2 n_3 n_4}^{(5)} \epsilon = 0. \end{aligned} \quad (3.4)$$

Here $\epsilon = \epsilon^1 + i\epsilon^2$ and ϵ^1, ϵ^2 are both Majorana-Weyl spinors with

$$\Gamma_{11} \epsilon = -\epsilon \quad (3.5)$$

where $\Gamma_{11} = \Gamma_0 \Gamma_1 \dots \Gamma_9$. Note that the integrability conditions for supersymmetry, $[\hat{\nabla}_m, \hat{\nabla}_n] \epsilon = 0$, imply that

$$\begin{aligned} &\left[R_{mns_1 s_2} - \frac{1}{48} F_{ms_1 r_1 r_2 r_3}^{(5)} F_{ns_2}{}^{r_1 r_2 r_3} \right] \Gamma^{s_1 s_2} \epsilon \\ &+ \left[\frac{i}{24} \nabla_{[m} F_{n] s_1 s_2 s_3 s_4}^{(5)} + \frac{1}{96} F_{mnr_1 r_2 s_1}^{(5)} F_{s_2 s_3 s_4}^{(5) r_1 r_2} \right] \Gamma^{s_1 s_2 s_3 s_4} \epsilon = 0 \end{aligned} \quad (3.6)$$

⁴Note a typo in the factor appearing in the Chern-Simons term in [3]: see also [36].

where in obtaining this expression we have made use of the identity

$$F^{(5)r_1 r_2 r_3}{}_{m_1[m_2} F^{(5)}{}_{m_3 m_4]r_1 r_2 r_3} = 0 . \quad (3.7)$$

To calculate the preserved supersymmetries we will use the following, slightly non-obvious, frame. We take e^0, e^1, e^2, e^3, e^4 to be a frame for the five-dimensional solution given by

$$\begin{aligned} e^0 &= dt + \frac{r^2}{2l} \sigma_3^L + \frac{fr^2}{V} \sigma_1^L, & e^1 &= \frac{1}{V^{1/2}} dr, \\ e^2 &= \frac{r}{2} \sigma_1^L, & e^3 &= \frac{r}{2} \sigma_2^L, & e^4 &= \frac{r}{2} V^{1/2} \sigma_3^L, \end{aligned} \quad (3.8)$$

and supplement this with

$$\begin{aligned} e^5 &= l d\alpha, & e^6 &= l \cos \alpha d\beta, \\ e^7 &= l \sin \alpha \cos \alpha [d\xi_1 - \sin^2 \beta d\xi_2 - \cos^2 \beta d\xi_3], \\ e^8 &= l \cos \alpha \sin \beta \cos \beta [d\xi_2 - d\xi_3], \\ e^9 &= -\frac{2}{\sqrt{3}} A - l \sin^2 \alpha d\xi_1 - l \cos^2 \alpha (\sin^2 \beta d\xi_2 + \cos^2 \beta d\xi_3) \end{aligned} \quad (3.9)$$

where $A = (\sqrt{3}/2)(fr^2/V)\sigma_1^L$ for the solution under consideration. Thus we have

$$\begin{aligned} F^{(5)} &= -4l^{-1} [e^0 \wedge e^1 \wedge e^2 \wedge e^3 \wedge e^4 + e^5 \wedge e^6 \wedge e^7 \wedge e^8 \wedge e^9] \\ &\quad + \frac{2}{\sqrt{3}} (e^5 \wedge e^7 + e^6 \wedge e^8) \wedge (*_{(5)} F^{(2)} - e^9 \wedge F^{(2)}) \\ &= -4l^{-1} [e^0 \wedge e^1 \wedge e^2 \wedge e^3 \wedge e^4 + e^5 \wedge e^6 \wedge e^7 \wedge e^8 \wedge e^9] \\ &\quad - 4fV^{-3/2} (e^0 + e^9) \wedge (e^1 \wedge e^2 - e^3 \wedge e^4) \wedge (e^5 \wedge e^7 + e^6 \wedge e^8) \end{aligned} \quad (3.10)$$

where the first expression is valid for a general solution of minimal gauged supergravity. The spin connection components for the solution (2.1), (2.3) are contained in the appendix.

Using a computer algebra package, the constraints on the Killing spinor imposed by the integrability conditions can be obtained. In particular, in the above orthonormal frame, we find the constraints

$$\begin{aligned} \Gamma^{0149} \epsilon &= -i\epsilon, & \Gamma^{0239} \epsilon &= i\epsilon \\ \Gamma^{0579} \epsilon &= -i\epsilon, & \Gamma^{09} \epsilon &= \epsilon . \end{aligned} \quad (3.11)$$

Hence we observe that the solution preserves at most 1/16 of the supersymmetry, or two supersymmetries. Now, since the solution was obtained by uplifting a solution of five-dimensional minimal gauged supergravity that preserved 2 supersymmetries, the ten-dimensional solution is expected to preserve at least 2 supersymmetries and hence (3.11) indicates that it preserves precisely 2 supersymmetries. Indeed just using these constraints we can solve the Killing spinor equation to obtain the Killing spinors in explicit form. We find the simple expression

$$\epsilon = e^{\frac{i}{2}(\frac{3t}{l} - \xi_1 - \xi_2 - \xi_3)} \eta \quad (3.12)$$

where η is a constant spinor, satisfying the projections (3.11). Some useful formulae for verifying this result are presented in an appendix. Note that we have checked that the solution obtained by changing the sign of the five-form also preserves two supersymmetries. It is interesting to note that the projections are the same projections for three orthogonally intersecting D3-branes with momentum along the common intersection direction (the nine direction).

4 Supersymmetry of uplifted AdS_5 black holes

Asymptotically AdS_5 black hole solutions of minimal $D = 5$ gauged supergravity with regular horizons have recently been constructed in [31]. Here we uplift these solutions to obtain solutions of type IIB supergravity and show that they preserve two supersymmetries.

The metric of the five-dimensional solution is specified by the funfbein given by

$$\begin{aligned} e^0 &= \mathcal{F}(dt + \Psi\sigma_3^L) \\ e^1 &= \mathcal{F}^{-1}\left(1 + \frac{2\omega^2}{l^2} + \frac{r^2}{l^2}\right)^{-\frac{1}{2}}dr \\ e^2 &= \frac{r}{2}\sigma_1^L \\ e^3 &= \frac{r}{2}\sigma_2^L \\ e^4 &= \frac{r}{2l}\sqrt{l^2 + 2\omega^2 + r^2}\sigma_3^L \end{aligned} \quad (4.1)$$

where

$$\mathcal{F} = 1 - \frac{\omega^2}{r^2} \quad (4.2)$$

and

$$\Psi = -\frac{\eta r^2}{2l}\left(1 + \frac{2\omega^2}{r^2} + \frac{3\omega^4}{2r^2(r^2 - \omega^2)}\right) \quad (4.3)$$

where $\eta = \pm 1$ and ω is constant. The 2-form gauge potential is given by

$$A = \frac{\sqrt{3}}{2}\left[\mathcal{F}dt + \frac{\eta\omega^4}{4lr^2}\sigma_3^L\right]. \quad (4.4)$$

Hence the metric of the uplifted solution is obtained by taking the ten-dimensional frame e^0, e^1, e^2, e^3 and e^4 given above in (4.1) together with e^5, e^6, e^7, e^8, e^9 given in (3.9). Using the first line of (3.10) we deduce that the self-dual five-form field strength is given by

$$\begin{aligned} F^{(5)} &= -4l^{-1}(e^0 \wedge e^1 \wedge e^2 \wedge e^3 \wedge e^4 + e^5 \wedge e^6 \wedge e^7 \wedge e^8 \wedge e^9) \\ &+ \left[-\frac{\eta\omega^4}{lr^4}(e^0 \wedge e^1 \wedge e^4 - e^2 \wedge e^3 \wedge e^9) + \frac{\eta\omega^2}{lr^4}(2r^2 + \omega^2)(e^0 \wedge e^2 \wedge e^3 - e^1 \wedge e^4 \wedge e^9)\right. \\ &\left.+ \frac{2\omega^2}{lr^3}\sqrt{l^2 + 2\omega^2 + r^2}(e^0 \wedge e^1 \wedge e^9 + e^2 \wedge e^3 \wedge e^4)\right] \wedge (e^5 \wedge e^7 + e^6 \wedge e^8). \end{aligned} \quad (4.5)$$

As in the last section, by first examining the integrability conditions for supersymmetry (3.6), we find that they type IIB solution preserves exactly two supersymmetries. The explicit form of the Killing spinors is

$$\epsilon = \mathcal{F}^{\frac{1}{2}} e^{-\frac{i}{2}(\xi_1 + \xi_2 + \xi_3)} \epsilon_0 \quad (4.6)$$

where ϵ_0 is a constant spinor satisfying the constraints

$$\begin{aligned} \Gamma^{0149} \epsilon_0 &= i\eta \epsilon_0, & \Gamma^{0239} \epsilon_0 &= -i\eta \epsilon_0 \\ \Gamma^{0579} \epsilon_0 &= -i\epsilon, & \Gamma^{09} \epsilon_0 &= \epsilon_0. \end{aligned} \quad (4.7)$$

Note that when $\eta = -1$ these are precisely the same projections as (3.11).

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A Formulae for Killing spinor calculation I

For the deformation of $AdS_5 \times S^5$ solution, the spin connection for the frame (3.8), (3.9) can be calculated and used to obtain $D_\mu \epsilon$. We find:

$$\begin{aligned} \partial_t \epsilon &+ \frac{1}{2} \left[\frac{1}{l} (\Gamma_{14} - \Gamma_{23}) + \frac{2f}{V^{3/2}} (\Gamma_{12} - \Gamma_{34}) \right] \epsilon, \\ \partial_r \epsilon &- \frac{1}{2} \left[\frac{1}{lV^{1/2}} \Gamma_{04} + \frac{2f}{V^2} (\Gamma_{29} + \Gamma_{02}) \right] \epsilon, \\ \partial_\theta \epsilon &+ \frac{1}{2} \left[-\frac{r}{2l} (\cos \phi \Gamma_{02} - \sin \phi \Gamma_{03}) - \frac{V^{1/2}}{2} [\cos \phi (\Gamma_{13} - \Gamma_{24}) + \sin \phi (\Gamma_{12} + \Gamma_{34})] \right. \\ &\quad \left. + \frac{fr}{V^{3/2}} [\cos \phi (\Gamma_{04} + \Gamma_{49}) + \sin \phi (\Gamma_{01} + \Gamma_{19})] + \frac{fr^2}{lV} \sin \phi (\Gamma_{14} - \Gamma_{23} - \Gamma_{57} - \Gamma_{68}) \right] \epsilon, \\ \partial_\phi \epsilon &+ \frac{1}{2} \left[-\frac{1}{2} \Gamma_{23} + \frac{rV^{1/2}}{2l} \Gamma_{01} - \frac{V}{2} \Gamma_{14} + \frac{fr^2}{lV^{3/2}} (\Gamma_{12} - \Gamma_{34}) - \frac{fr}{V} (\Gamma_{03} + \Gamma_{39}) \right] \epsilon, \\ \partial_\psi \epsilon &+ \frac{1}{2} \left[-\frac{1}{2} \cos \theta \Gamma_{23} - \frac{r}{2l} \sin \theta (\cos \phi \Gamma_{03} + \sin \phi \Gamma_{02}) + \frac{rV^{1/2}}{2l} \cos \theta \Gamma_{01} \right. \\ &\quad \left. + \frac{1}{2} [\sin \theta \cos \phi (\Gamma_{34} + \Gamma_{12}) - \sin \theta \sin \phi (\Gamma_{13} - \Gamma_{24})] - \frac{V}{2} \cos \theta \Gamma_{14} \right. \\ &\quad \left. + \frac{fr}{V^{3/2}} \sin \theta [\cos \phi (-\Gamma_{19} - \Gamma_{01}) + \sin \phi (\Gamma_{04} + \Gamma_{49})] \right. \\ &\quad \left. - \frac{fr^2}{lV^{3/2}} \cos \theta (\Gamma_{34} - \Gamma_{12}) + \frac{fr}{V} \cos \theta (-\Gamma_{39} - \Gamma_{03}) \right. \\ &\quad \left. + \frac{fr^2}{lV} \sin \theta \cos \phi (\Gamma_{57} + \Gamma_{68} - \Gamma_{14} + \Gamma_{23}) \right] \epsilon, \\ \partial_\alpha \epsilon &- \frac{1}{2} \Gamma_{79} \epsilon, \\ \partial_\beta \epsilon &+ \frac{1}{2} \left[\sin \alpha (\Gamma_{56} + \Gamma_{78}) - \cos \alpha \Gamma_{89} \right] \epsilon, \end{aligned}$$

$$\begin{aligned}
\partial_{\xi_1} \epsilon &- \frac{1}{2} \left[\cos \alpha (\cos \alpha \Gamma_{57} - \sin \alpha \Gamma_{59}) + \frac{2fl}{V^{3/2}} \sin^2 \alpha (\Gamma_{12} - \Gamma_{34}) \right] \epsilon, \\
\partial_{\xi_2} \epsilon &+ \frac{1}{2} \left[-\sin \alpha \sin \beta (\sin \alpha \sin \beta \Gamma_{57} - \cos \beta \Gamma_{58} + \cos \alpha \sin \beta \Gamma_{59}) \right. \\
&\quad \left. + \cos \beta (\sin \alpha \sin \beta \Gamma_{67} - \cos \beta \Gamma_{68} + \cos \alpha \sin \beta \Gamma_{69}) \right. \\
&\quad \left. + \frac{2fl}{V^{3/2}} \cos^2 \alpha \sin^2 \beta (\Gamma_{34} - \Gamma_{12}) \right] \epsilon, \\
\partial_{\xi_3} \epsilon &+ \frac{1}{2} \left[-\sin \alpha \cos \beta (\sin \alpha \cos \beta \Gamma_{57} + \sin \beta \Gamma_{58} + \cos \alpha \cos \beta \Gamma_{59}) \right. \\
&\quad \left. - \sin \beta (\sin \alpha \cos \beta \Gamma_{67} + \sin \beta \Gamma_{68} + \cos \alpha \cos \beta \Gamma_{69}) \right. \\
&\quad \left. + \frac{2fl}{V^{3/2}} \cos^2 \alpha \cos^2 \beta (\Gamma_{34} - \Gamma_{12}) \right] \epsilon. \tag{A.1}
\end{aligned}$$

In determining the Killing spinors it is also useful to note

$$\frac{i}{1920} \Gamma^{n_1 n_2 n_3 n_4 n_5} F_{n_1 n_2 n_3 n_4 n_5}^{(5)} = \frac{i}{4} \left[\frac{1}{l} \Gamma_{01234} + \frac{f}{V^{3/2}} \Gamma_0 (\Gamma_{57} + \Gamma_{68}) (\Gamma_{12} - \Gamma_{34}) \right] (1 + \Gamma_{11}). \tag{A.2}$$

B Formulae for Killing spinor calculation II

The formula for $D_\mu \epsilon$ for the frame (4.1), (3.9) of the uplifted AdS black holes is given by

$$\begin{aligned}
\partial_t \epsilon &+ \frac{1}{2} \left[\sqrt{l^2 + r^2 + 2\omega^2} \frac{\omega^2}{lr^3} \mathcal{F} (\Gamma_{01} + \Gamma_{19}) - \frac{\eta}{l} \mathcal{F}^2 \Gamma_{14} + \frac{(r^2 + \omega^2)\eta}{r^2 l} \mathcal{F} \Gamma_{23} \right. \\
&\quad \left. - \frac{\mathcal{F}}{l} (\Gamma_{57} + \Gamma_{68}) \right] \epsilon, \\
\partial_r \epsilon &+ \frac{1}{2} \left[-\frac{\eta(4\omega^2 r^2 + \omega^4 - 2r^4)}{2r^4 \sqrt{l^2 + r^2 + 2\omega^2} \mathcal{F}} \Gamma_{04} + \frac{\omega^2}{r^3 \mathcal{F}} \Gamma_{09} - \frac{\eta\omega^2(2r^2 + \omega^2)}{2r^4 \sqrt{l^2 + r^2 + 2\omega^2} \mathcal{F}} \Gamma_{49} \right] \epsilon, \\
\partial_\theta \epsilon &+ \frac{1}{2} \left[\frac{\eta}{4r^3 l} (-2r^4 - 2r^2 \omega^2 + \omega^4) (-\cos \phi \Gamma_{02} + \sin \phi \Gamma_{03}) \right. \\
&\quad \left. - \frac{1}{2l} \mathcal{F} \sqrt{l^2 + r^2 + 2\omega^2} (\sin \phi \Gamma_{12} + \cos \phi \Gamma_{13}) \right. \\
&\quad \left. + \frac{1}{2l} \sqrt{l^2 + r^2 + 2\omega^2} (\cos \phi \Gamma_{24} - \sin \phi \Gamma_{34}) - \frac{\eta\omega^4}{4r^3 l} (\cos \phi \Gamma_{29} - \sin \phi \Gamma_{39}) \right] \epsilon, \\
\partial_\phi \epsilon &+ \frac{1}{2} \left[\frac{\eta}{4r^5 l^2} \sqrt{l^2 + r^2 + 2\omega^2} (-3\omega^4 r^2 + \omega^6 - 2r^6) \Gamma_{01} \right. \\
&\quad \left. - \frac{1}{2r^2 l^2} (r^2 \omega^2 + \omega^4 + r^4 + r^2 l^2 - \omega^2 l^2) \Gamma_{14} - \frac{\eta\omega^4}{4r^3 l^2} \sqrt{l^2 + r^2 + 2\omega^2} \mathcal{F} \Gamma_{19} \right. \\
&\quad \left. + \frac{1}{2r^4 l^2} (\omega^6 - r^4 l^2) \Gamma_{23} - \frac{\eta\omega^4}{4l^2 r^2} (\Gamma_{57} + \Gamma_{68}) \right] \epsilon \\
\partial_\psi \epsilon &+ \frac{1}{2} \left[\frac{\eta}{4r^5 l^2} \cos \theta \sqrt{l^2 + r^2 + 2\omega^2} (-3\omega^4 r^2 + \omega^6 - 2r^6) \Gamma_{01} \right. \\
&\quad \left. - \frac{\eta}{4r^3 l} \sin \theta (-2r^4 - 2r^2 \omega^2 + \omega^4) (\sin \phi \Gamma_{02} + \cos \phi \Gamma_{03}) \right] \epsilon
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2l} \sin \theta \sqrt{l^2 + r^2 + 2\omega^2} \mathcal{F} (\cos \phi \Gamma_{12} - \sin \phi \Gamma_{13}) \\
& - \frac{1}{2r^2 l^2} \cos \theta (r^2 \omega^2 + \omega^4 + r^4 + r^2 l^2 - \omega^2 l^2) \Gamma_{14} - \frac{\eta}{4l^2 r^2} \omega^4 \cos \theta (\Gamma_{57} + \Gamma_{68}) \\
& - \frac{\eta}{4r^3 l^2} \cos \theta \sqrt{l^2 + r^2 + 2\omega^2} \omega^4 \mathcal{F} \Gamma_{19} + \frac{1}{2r^4 l^2} \cos \theta (\omega^6 - r^4 l^2) \Gamma_{23} \\
& + \frac{1}{2l} \sin \theta \sqrt{l^2 + r^2 + 2\omega^2} (\sin \phi \Gamma_{24} + \cos \phi \Gamma_{34}) - \frac{\eta}{4r^3 l} \sin \theta \omega^4 (\sin \phi \Gamma_{29} + \cos \phi \Gamma_{39}) \Big] \epsilon \\
\partial_\alpha \epsilon & - \frac{1}{2} \Gamma_{79} \epsilon \\
\partial_\beta \epsilon & + \frac{1}{2} [\sin \alpha (\Gamma_{56} + \Gamma_{78}) - \cos \alpha \Gamma_{89}] \epsilon \\
\partial_{\xi_1} \epsilon & + \frac{1}{2} \Big[-\frac{\omega^2}{r^3} \sin^2 \alpha \sqrt{l^2 + r^2 + 2\omega^2} \Gamma_{01} - \frac{\eta}{2r^4} \omega^2 \sin^2 \alpha (2r^2 + \omega^2) \Gamma_{14} \\
& + \frac{\eta}{2r^4} \omega^4 \sin^2 \alpha \Gamma_{23} - \cos^2 \alpha \Gamma_{57} + \cos \alpha \sin \alpha \Gamma_{59} \Big] \epsilon \\
\partial_{\xi_2} \epsilon & + \frac{1}{2} \Big[-\frac{\omega^2}{r^3} \cos^2 \alpha \sin^2 \beta \sqrt{l^2 + r^2 + 2\omega^2} \Gamma_{01} - \frac{\eta \omega^2}{2r^4} \cos^2 \alpha \sin^2 \beta (2r^2 + \omega^2) \Gamma_{14} \\
& + \frac{\eta \omega^4}{2r^4} \cos^2 \alpha \sin^2 \beta \Gamma_{23} - \sin^2 \beta \sin^2 \alpha \Gamma_{57} + \sin \beta \cos \beta \sin \alpha (\Gamma_{58} + \Gamma_{67}) \\
& - \cos \alpha \sin \alpha \sin^2 \beta \Gamma_{59} - \cos^2 \beta \Gamma_{68} + \cos \alpha \sin \beta \cos \beta \Gamma_{69} \Big] \epsilon \\
\partial_{\xi_3} \epsilon & + \frac{1}{2} \Big[-\frac{l \omega^2}{r^3} \cos^2 \alpha \cos^2 \beta \sqrt{l^2 + r^2 + 2\omega^2} \Gamma_{01} - \frac{\eta l \omega^2}{2r^4} (2r^2 + \omega^2) \cos^2 \alpha \cos^2 \beta \Gamma_{14} \\
& + \frac{\eta l \omega^4}{2r^4} \cos^2 \alpha \cos^2 \beta \Gamma_{23} - l \cos^2 \beta \sin^2 \alpha \Gamma_{57} - l \sin \alpha \sin \beta \cos \beta (\Gamma_{58} + \Gamma_{67}) \\
& - l \cos \alpha \sin \alpha \cos^2 \beta \Gamma_{59} - l \sin^2 \beta \Gamma_{68} - l \cos \alpha \sin \beta \cos \beta \Gamma_{69} \Big] \epsilon .
\end{aligned} \tag{B.1}$$

C Ambiguities in the energy-momentum tensor

Using signature $(-, +, +, +)$ and standard conventions for curvature tensors, the expressions for the tensors obtained from the variation of (2.17) and given in [37], become

$$\begin{aligned}
^{(1)}H_{\mu\nu} &= 2\nabla_\mu \nabla_\nu R - 2g_{\mu\nu} \square R + \frac{1}{2} g_{\mu\nu} R^2 - 2R R_{\mu\nu}, \\
^{(2)}H_{\mu\nu} &= \nabla_\mu \nabla_\nu R - \frac{1}{2} g_{\mu\nu} \square R - \square R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R^{\alpha\beta} R_{\alpha\beta} - 2R^{\alpha\beta} R_{\alpha\mu\beta\nu}, \\
H_{\mu\nu} &= \frac{1}{2} g_{\mu\nu} R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} - 2R_{\mu\alpha\beta\gamma} R_\nu^{\alpha\beta\gamma} - 4\square R_{\mu\nu} + 2\nabla_\mu \nabla_\nu R \\
&\quad + 4R_{\mu\alpha} R^\alpha_\nu - 4R^{\alpha\beta} R_{\alpha\mu\beta\nu} .
\end{aligned} \tag{C.1}$$

These are each conserved and satisfy $H = -^{(1)}H + 4^{(2)}H$. For the deformed AdS_5 background one finds that $^{(1)}H_{\mu\nu}$ and $^{(2)}H_{\mu\nu}$ can be written in terms of the orthonor-

mal frame (2.15) as

$${}^{(1)}H_{ab} = -6 \begin{bmatrix} 3 & -x & 0 & 0 \\ -x & 1 & 0 & -x \\ 0 & 0 & 1 & 0 \\ 0 & -x & 0 & 1 \end{bmatrix} \quad (\text{C.2})$$

$${}^{(2)}H_{ab} = -2 \begin{bmatrix} 3 + \frac{3}{4}x^2 & -x & 0 & \frac{3}{4}x^2 \\ -x & 1 & 0 & -x \\ 0 & 0 & 1 & 0 \\ \frac{3}{4}x^2 & -x & 0 & 1 + \frac{3}{4}x^2 \end{bmatrix} \quad (\text{C.3})$$

The contribution coming from (2.19) is also easily calculated. The contribution coming from $\gamma_{ab}F^2$ vanishes while the contribution coming from $F_{ac}F_b{}^c$ gives

$$-\frac{1}{8\pi G} \frac{3l^3\delta}{4} \begin{bmatrix} x^2 & 0 & 0 & x^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ x^2 & 0 & 0 & x^2 \end{bmatrix} \quad (\text{C.4})$$

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